

Implications of the unusual structure in the pp correlation from Pb+Pb collisions at 158 AGeV

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The recent NA49 measurement of two-proton correlation function shows an interesting and unexpected structure at large relative momentum. Applying source imaging techniques to the measurement, we find an unusually steep drop-off in the two-proton source function. We show that the steep drop-off is due to the structure in the correlation and the drop-off cannot be explained using conventional correlation analysis. We suggest possible physics reasons for the unusual source function.

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The NA49 collaboration has recently measured the two-proton correlation function from Pb+Pb collisions at 158 AGeV, integrated over transverse momentum in the rapidity range $2.4 < y < 3.4$ (midrapidity ≈ 2.9) [1]. The measurement shows an interesting structure around q_{inv} (the magnitude of the proton momentum in the pair center-of-mass frame) of 70 MeV/c, which statistically significantly deviates from the expected unity. NA49 has extensively studied possible systematic effects and none of these systematics can account for the structure. In this letter, we pursue possible physics explanations for the structure.

Similar structure has not been seen in other experiments for different collision systems or energies. Similar structures have been predicted in the tails of two-pion correlations [2,3] and have been ascribed to various effects, from hidden correlations [2] to a breakdown of the smoothness approximation [3]. Applying these predictions to two-proton correlations is difficult because of the differences between the two-proton and two-pion final state interactions. Source imaging [4,5] is an ideal tool to separate effects due to two-proton final state interactions and wavefunction antisymmetrization from those due to the source function itself. Moreover, one can obtain the source function in a model-independent manner, i.e. without a Gaussian source assumption.

We have inverted the NA49 two-proton correlation function and obtained a two-proton source function that is exceptional in several ways. Our naive expectation was that the source would appear somewhat Gaussian (as is the case for all two-pion and two-proton sources that have been obtained so far [4-8]). Instead, the source is consistent with a step-function with radius ~ 10 fm. In some

sense this is not a surprise as the structure in the tail of the NA49 correlation function is reminiscent of the Fourier transform of a sharp object. In fact, the edge can be removed by smoothing out the structure. As if finding a step-function source is not strange enough, *this source cannot be explained within the standard Koonin-Pratt formalism*: in the Koonin-Pratt formalism, the Fourier transform of a source must be positive everywhere while the Fourier transform of the NA49 source is not. Thus, using imaging and this test of positivity, one may infer whether a given source is breaking *something*. Knowing this, the task then becomes understanding what is broken.

The outline of this letter is as follows. First, we describe the conventional expectations for sources in the Koonin-Pratt formalism. Second, we detail why the Fourier transform of a source in the Koonin-Pratt formalism must be positive, thus giving us a test to see if a given source fits in this formalism. Then, we perform the inversion of the NA49 two-proton correlation function and show that it fails this test. We demonstrate that the structure in the tail of the correlation function is the origin of the failure. Finally, we discuss physics that might be missing from the Koonin-Pratt formalism that could account for the unusual behavior of the source.

We begin with our expectations for the source function in the Koonin-Pratt formalism [9-11]. A two-particle correlation function can be written as a convolution of a source function, $S_{q'}(\vec{r})$, with the Wigner transform of the particle pair wavefunction, $g_{\vec{q}}(\vec{r}, q')$, in the pair center-of-mass system (c.m.s.) [11]:

$$C(\vec{q}) = \int d^3r \frac{d^4q'}{(2\pi)^4} g_{\vec{q}}(\vec{r}, q') S_{q'}(\vec{r}). \quad (1)$$

Here $\vec{q} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ is the asymptotic particle momentum, q' is the internal four-momentum (and need not correspond to on-shell particle momenta), and $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the separation of emission points. Both $g_{\vec{q}}(\vec{r}, q')$ and q' are defined through

$$g_{\vec{q}}(\vec{r}, q') = \int d^4\zeta e^{i\zeta \cdot q'} \Phi_{\vec{q}}^*(r + \zeta/2) \Phi_{\vec{q}}(r - \zeta/2). \quad (2)$$

Here $\Phi_{\vec{q}}(r) = \phi_{\vec{q}}(\vec{r})e^{-iq_0 r_0}$ is the particle pair wavefunction. Note that the different phases of the wavefunction in Eq. (2) make $g_{\vec{q}}$ independent of the relative emission time.

In the Koonin-Pratt formalism, one makes two assumptions. The first is that the source function only has a weak dependence on relative momentum corresponding to off-shell particles, i.e. $S_{q'}(\vec{r}) \approx S_{\vec{q}}(\vec{r})$. This is a *smoothness assumption* [3,12]. With this, the integral over the internal momentum q' in Eq. (1) can be performed and we obtain

$$C(\vec{q}) = \int d^3r |\phi_{\vec{q}}(\vec{r})|^2 S_{\vec{q}}(\vec{r}). \quad (3)$$

One further assumes that *the two-particle source is a convolution of single-particle sources* (i.e. particles are emitted independently):

$$S_{\vec{q}}(\vec{r}) = \int dt_1 dt_2 d^3R D\left(\vec{R} + \frac{\vec{r}}{2}, t_1; \vec{q}\right) D\left(\vec{R} - \frac{\vec{r}}{2}, t_2; -\vec{q}\right), \quad (4)$$

where D is the normalized single-particle source in the c.m.s. and has the conventional interpretation as the normalized production rate of final-state particles. The integral of D over the particle emission times, $\int dt D(\vec{r}, t; \vec{q})$, is then the normalized phase-space distribution of particles after last collision (freeze-out) and $S_{\vec{q}}(\vec{r})$ is the probability density for a pair each with momentum \vec{q} to be emitted a distance \vec{r} apart in the c.m.s.

In Eq. (4) we do not need to consider the contribution to the source from pairs with large relative momentum ($|\vec{q}| = q \gtrsim q_{\text{cut}} \approx 100 \text{ MeV}/c$): the kernel cuts off the contribution from these pairs. This happens because the source varies slowly on the length scale of the oscillations in the kernel for $q \gtrsim q_{\text{cut}}$, hence the integral in Eq. (3) averages to zero. The fact that only low- q pairs contribute to the correlation is used to justify dropping the q dependence from the single-particle sources entirely [10,13].

The Fourier transform of Eq. (4) is

$$\begin{aligned} \tilde{S}(\vec{k}) &= \int d^3r e^{i\vec{r} \cdot \vec{k}} S_{\vec{q}}(\vec{r}) = \int d^3R d^3r e^{i\vec{r} \cdot \vec{k}} \times \\ &\int dt_1 D\left(\vec{R} + \frac{\vec{r}}{2}, t_1; \vec{q}\right) \int dt_2 D\left(\vec{R} - \frac{\vec{r}}{2}, t_2; -\vec{q}\right) \\ &\equiv \tilde{D}(\vec{k}, \vec{q}) \tilde{D}^*(\vec{k}, -\vec{q}), \end{aligned} \quad (5)$$

where \tilde{D} is the Fourier transform of the time-integrated single-particle source. *If we can neglect the \vec{q} dependence of the source function* (or equivalently the momentum dependence of the single-particle sources in the c.m.s.), then we find the following interesting result:

$$\tilde{S}(\vec{k}) = \int d^3r e^{i\vec{r} \cdot \vec{k}} S(\vec{r}) = |\tilde{D}(\vec{k})|^2 \geq 0. \quad (6)$$

So, checking the Fourier transform of the source for positivity can serve as a test of the validity of the underlying assumptions.

For pions, we note that Eq. (6) is actually a restatement of the theorem that the correlation function must always be larger than one (after the Coulomb correction). To see this, first realize that sources of indistinguishable particles must be symmetric under coordinate reversion: $\vec{r} \rightarrow -\vec{r}$. Using this, Eq. (6) becomes $\tilde{S}(\vec{k}) = \int d^3r \cos(\vec{r} \cdot \vec{k}) S(\vec{r}) \geq 0$. Now, for like-pions Eq. (3) gives $C(\vec{q}) - 1 = \int d^3r \cos(2\vec{r} \cdot \vec{q}) S(\vec{r}) = \tilde{S}(2\vec{q})$ which must be positive.

We now turn to the practical issues of extracting information about the source from the NA49 correlation function [1]. The NA49 correlation function is averaged over the orientation of \vec{q} so we integrate out the angular dependence of the correlation, arriving at

$$C(q) - 1 = 4\pi \int dr r^2 K(q, r) S_{\vec{q}}(r). \quad (7)$$

Here $q = |\vec{q}|$ and $r = |\vec{r}|$. The angle averaged kernel, $K(q, r) = \int d(\cos \theta_{\vec{q}\vec{r}}) |\phi_{\vec{q}}(\vec{r})|^2 - 1$, can be written as a sum over the two-proton partial waves:

$$K(q, r) = \frac{1}{2} \sum_{js\ell\ell'} (2j+1) \left(g_{js}^{\ell\ell'}(r)\right)^2 - 1. \quad (8)$$

In what follows, we calculate the two-proton relative wavefunctions by solving the Schrödinger equation with the REID93 [14] and Coulomb potentials for partial waves with $\ell \leq 2$ and use pure Coulomb waves for those with $\ell \geq 3$. Note that the results are not noticeably altered by using the full solutions to the Schrödinger equation for the higher partial waves due to the low relative momentum of the pair.

Since both Eqs. (3) and (7) are simple integral equations with non-singular kernels, they may be inverted to obtain the source function. We begin by discretizing either Eq. (3) or (7) to obtain the matrix equation $C_i = \sum_j K_{ij} S_j$. If the data were measured with infinite precision we could invert this matrix equation to find $S_j = \sum_i (K^{-1})_{ij} C_i$. Since we must account for the finite measurement errors, we proceed as in [4–7] and find the set of source points, S_j , that minimize the χ^2 . Here, $\chi^2 = \sum_i (C_i - \sum_j K_{ij} S_j)^2 / \Delta^2 C_i$. This source is $S_j = \sum_i [(K^T B K)^{-1} K^T B]_{ji} (C_i - 1)$ where K^T is the transpose of the kernel matrix and B is the inverse covariance matrix of the data $B_{ij} = \delta_{ij} / \Delta^2 C_i$. The error on

the source is the square-root of the diagonal elements of the covariance matrix of the source, $\Delta^2 S = (K^T B K)^{-1}$.

In this inversion process, all q dependence of the correlation is ascribed to the kernel. If only the kernel has a q dependence, then the imaging of Eq. (3) or (7) is unique. On the other hand, if the source also has a q dependence, then the imaging gives the source averaged over q . While only pairs with $q \lesssim q_{\text{cut}}$ contribute to the correlation, this does raise the possibility that a strong \bar{q} dependence might cause trouble with the imaging. This possibility will be investigated in a future article [15].

Averaging both sides of Eq. (6) over angle, we obtain a condition for the angle averaged source (which we assume to have no q dependence):

$$\tilde{S}(k) \equiv 4\pi \int_0^\infty dr r^2 S(r) j_0(kr) \geq 0, \quad (9)$$

where $j_0(kr)$ is the 0th order spherical Bessel function. The Fourier transform is a linear operation, so the errors propagate via $\Delta \tilde{S}(k) = 4\pi \int_0^\infty dr r^2 \Delta S(r) j_0(kr)$.

We now turn to the NA49 two-proton correlation data [1]. In Fig. 1a, we reproduce the data with the filled circles. Note the oscillation at $q \sim 70$ MeV/c and maybe at ~ 100 MeV/c. These oscillations are responsible for all the interesting behavior we discuss below. Figure 2a shows the source function obtained by imaging the correlation function in Fig. 1a. No constraints have been imposed in the imaging. Note that the source is nearly flat up to $r \simeq 10$ fm, after which the source drops rapidly to zero in the bin centered at 14 fm. The source remains several orders of magnitude smaller than the main part of the source out to the edge of the image at $r \approx 65$ fm. For comparison, we plot NA49's best-fit source in the dashed curve in Fig. 2a. NA49 fit a Gaussian single-particle source to the $q < 48$ MeV/c points of the correlation function and found a best-fit Gaussian radius of 3.85 fm. Examining Fig. 2a., the best-fit Gaussian source describes the $r \leq 10$ fm points and fails beyond that. As a side benefit of imaging, we can extract the integral of the source over the imaged region. We find $4\pi \int_0^{12\text{fm}} dr r^2 S(r) = 0.78 \pm 0.55$. We should mention that the constant weak decay correction applied in ref. [1] does increase the magnitude, and hence the integral, of the source over the uncorrected one, but the correction does not change the structure of the oscillation.

As a consistency check, we uninverted the source into the correlation function which is shown as the solid histogram in Fig. 1a. The resulting correlation function is consistent with the original but the structure in the tail of the correlation has been washed out. This is easily understood as an effect of the loss of information in going from a correlation function with ~ 20 independent data points to a source with only 7 points.

Now we test whether the imaged source fails the Fourier transform test discussed above. We must take

care in doing the integral in Eq. (9) as simply discretizing it introduces numerical error at high k , exactly where a failure of the Fourier transform test will occur. Instead, we use an interpolation procedure from ref. [16] to evaluate Eq. (9) with the $S(r)$ in Fig. 2a. The result of this integration is shown in Fig. 3a. $\tilde{S}(k)$ goes negative in the range 100–150 MeV/c.

For comparison, in Fig. 1b. we replace the structure in the tail of the correlation with statistical noise. To be exact, we replace the measured correlation function in $36 < q < 102$ MeV/c with a correlation consistent with one, but containing the original errors and statistical scatter. Applying the imaging technique, we obtain the source function shown in Fig. 2b. For comparison, we plot the best-fit Gaussian source along with the “smoothed” source in Fig. 2b and the two sources are consistent. Furthermore, not only does this modified source appear less flat and somewhat Gaussian but its Fourier transform is positive as shown in Fig. 3b. Thus we have demonstrated that the structure in the tail of the correlation is the cause of the failure shown in Fig. 3a. This assertion should not come as a surprise as the behavior of the correlation at the scale q is responsible for source structure on the scale $r \sim \pi \hbar c / q$.

At this point we comment on the model comparisons by NA49 using different single-particle and two-particle sources [1]. First, NA49 compared their data to correlations generated from VENUS and RQMD simulations. Both models generate the final freeze-out distribution of protons (equivalent to our $D(\vec{r}, t; \vec{q})$) and these distributions are input to a correlation afterburner that performs the integrals in Eqs. (4) and (3). Thus, by construction these correlations pass the Fourier transform test and cannot describe the tail of the correlation data. NA49 also tried a hard edged two-proton source but could not reproduce the observed structure in the correlation without abandoning the independent source assumption.

We are now left with a dilemma: something is wrong with either the imaging, the assumptions in the Koonin-Pratt formalism or both. While experimental confirmation of this new data is needed, we suggest three possible physics sources of the problem (as we have indicated in *italic text* in this letter): (1) a strong low- q dependence of the source that is not imaged, (2) the source not being a convolution of single-particle sources, or (3) a breakdown of the smoothness assumption.

In the case that there is a strong low- q dependence in the NA49 source function, such a q dependence cannot be reconstructed by the imaging. In this case, the Fourier transform test is inappropriate as the imaged source does not reflect the true NA49 source. One would think that a strong low- q dependence of the source might result from strong position-momentum correlations in the single-particle sources. In ref. [17], the authors deliberately increased the degree of position-momentum correlation by aligning the proton transverse momentum

with the transverse position of the proton at freeze-out in RQMD. Even after this dramatic increase in the position-momentum correlation, they were able to image the source function reliably.

In the second case, the NA49 two-proton source is not a convolution of two independent sources. In this case, both imaging and the Fourier test are appropriate. However, interpreting the source function becomes difficult as we cannot model it simply with a transport model. Makhlin and Surdutovich [2] suggest that this possibility could occur if there are hidden correlations in the system. Such hidden correlations would arise from having three or more dynamically or statistically correlated particles in the final state while only observing two of them.

Finally, in the event that the smoothness assumption [3] is no longer valid, the entire formalism breaks down as we can no longer perform the integral over the internal four-momentum in Eq. (1). This is the worst possible case from our standpoint as neither Eq. (3) nor Eq. (7) is valid and we cannot apply our imaging procedure. Pratt [3] argues that this could be caused by strong off-shell effects and these effects would be magnified by strong position-momentum correlations in the source and by the short-range interaction between the protons. Pratt comments that these effects are larger in small systems, e.g. with single-particle source size $R \sim 1$ fm, much smaller than the source imaged here.

In conclusion, we have imaged the recently measured NA49 two-proton correlation function and found an extraordinary two-proton source function. This source function appears to be a step function and we show that the steep fall-off in the source is due to the structure in the tail of the NA49 correlation function. Furthermore, we show that this source does not fit into the Koonin-Pratt formalism. While we have suggested several possible physics explanations for the structure in the two-proton correlation, its origin remains a puzzle.

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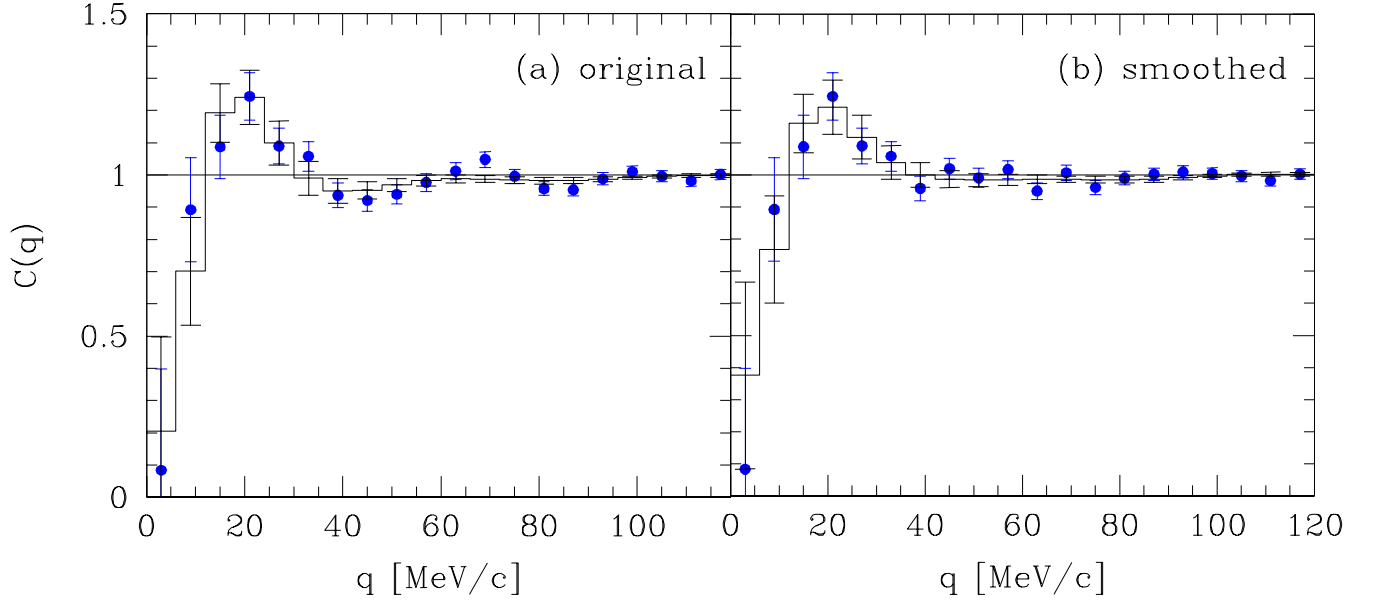


FIG. 1. (a) Correlation function from ref. [1] (filled circles, small caps on error bars) and the restored correlation function (solid histogram, large caps on error bars). (b) Same as (a) but the structure between $36 < q < 102$ MeV/c is replaced by a flat correlation with experimental error and realistic statistical scatter (filled circles, small caps on error bars), and the restored correlation function (solid histogram, large caps on error bars).

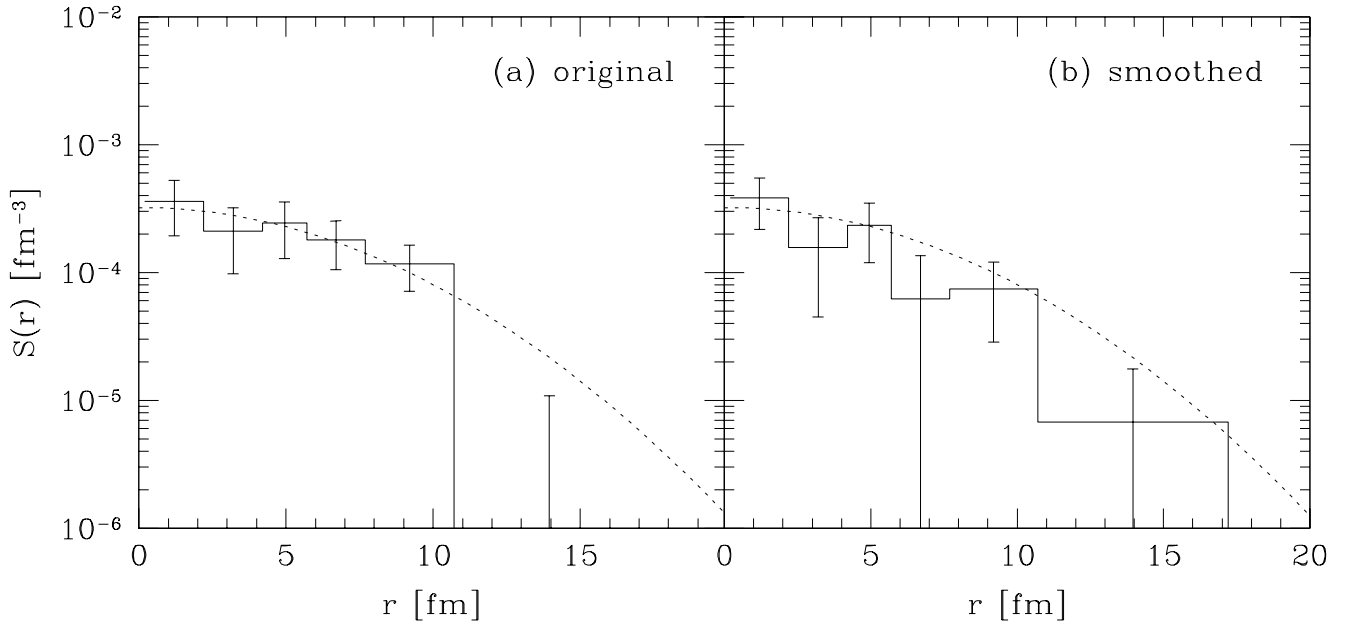


FIG. 2. (a) Source function obtained by imaging the correlation data (solid histogram) and NA49's best-fit Gaussian source (dashed curve). (b) Source function obtained by imaging the "smoothed" correlation function (solid histogram) also with NA49's best-fit Gaussian source (dashed curve).

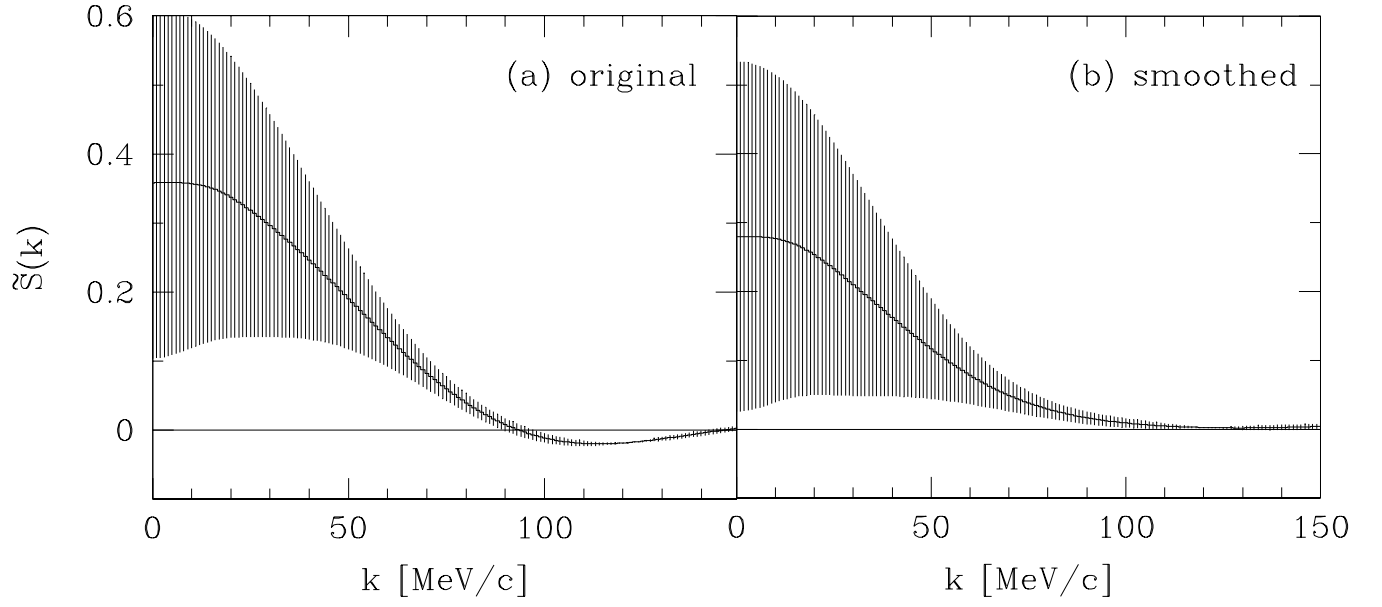


FIG. 3. (a) Fourier transform of the source function in Fig. 2a. Notice the dip below zero in the range $100 < k < 150$ MeV/c. (b) Fourier transform of the source function in Fig. 2b. Notice that this function is everywhere positive. In both panels, the vertical lines represent the error band of the calculation.